# Eddy Diffusivity Close to a Wall

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Son and Hanratty (3) have recently considered the limiting relation for eddy diffusivity for mass close to a wall in fully developed pipe flow. Mass transfer data for the range of Schmidt numbers from 400 to 2,400 were used to obtain the relation

$$\frac{\epsilon}{\nu} = 0.00032\ y^{+4} \tag{1}$$

for the eddy diffusivity for mass close to the wall. This relation is obtained from the equation

$$\frac{\epsilon}{\nu} = b(N_{Sc})y^{+n} \tag{2}$$

which assumes that the concentration boundary layer is thin enough so that only the first term in the Taylor series expansion of the relation

$$\frac{\epsilon}{\nu} = g(y^{+}) \tag{3}$$

need be considered. An equation for the dimensionless mass transfer coefficient was also derived for a thin concentration boundary layer:

$$K_\infty{}^{+} = \frac{n}{\pi} b^{1/n} \sin\left(\frac{\pi}{n}\right) (N_{Sc})^{-\left(\frac{n-1}{n}\right)} \tag{4}$$

The thickness of the concentration boundary layer decreases as the Schmidt number increases; therefore, high Schmidt number mass transfer data are of particular interest. Harriott and Hamilton (1) have obtained data for 2.37-cm. diameter pipe sections of benzoic acid dissolving in glycerin-water solutions and methocel solutions with Schmidt numbers up to 100,000 and Reynolds numbers above 10,000. Kishinevsky, Denisova, and Parmenov (2) have also obtained turbulent data for 1.3-cm. diameter sections of benzoic acid dissolving in glycerin-water and sucrose-water solutions with Schmidt numbers up to 31,000. It is interesting to consider the relationships presented by Son and Hanratty with respect to these high Schmidt number data.

Figure 1 shows the Harriott and Hamilton data and the Kishinevsky et al. data for the dimensionless mass transfer coefficient plotted as a function of Schmidt number. These data show different slopes from those obtained from the data treated by Son and Hanratty. These are

$<K_\infty{}^{+}> \sim (N_{Sc})^{-0.635}$ for Harriott and Hamilton and
$<K_\infty{}^{+}> \sim (N_{Sc})^{-0.57}$ for Kishenevsky et al.

Son and Hanratty obtained a $-3/4$ exponent for the Schmidt number. The Harriott and Hamilton data for a Schmidt number greater than 5,000 can be approximately represented by

$$K_\infty{}^{+} = 0.0816(N_{Sc})^{-2/3} \tag{5}$$

which indicates that $n = 3$. Substitution of $n = 3$ in Equation (4) and solution with Equation (5) gives the relation for the eddy diffusivity:

$$\frac{\epsilon}{\nu} = 0.00096\ y^{+3} \tag{6}$$

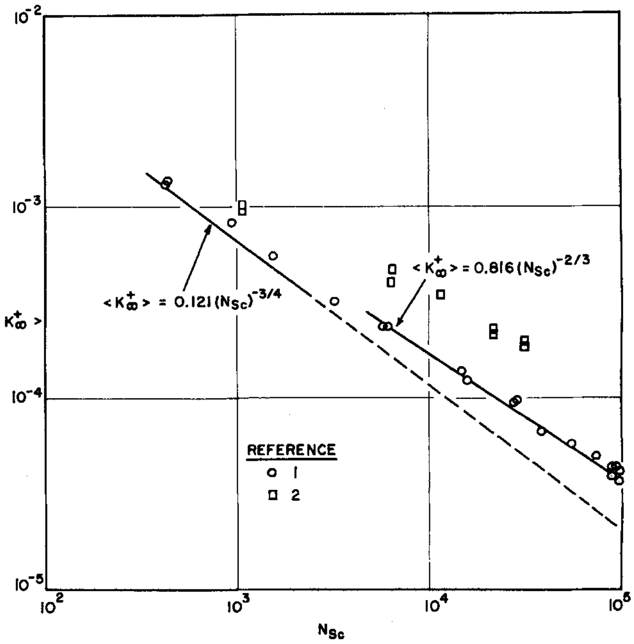


Fig. 1. Experimental $K_\infty{}^{+}$ for high Schmidt numbers.

The Harriott and Hamilton data appear to be more consistent with the data used by Son and Hanratty, and the data of Kishenevsky et al. appear to show unusually high mass transfer coefficients. Both sets of data do indicate that $n$ is more nearly equal to 3 than to 4 for a thin concentration boundary layer.

## NOTATION

$K$ = local mass transfer coefficient
$K^{+}$ = $K/u^{*}$
$<K>$ = average mass transfer coefficient over a section of wall
$K_\infty$ = limiting value of $K$ attained in the fully developed region
$N_{Sc}$ = Schmidt number
$u^{*}$ = friction velocity, $(\tau_w/\rho)^{1/2}$
$y$ = distance perpendicular to wall
$y^{+}$ = $yu^{*}/N$
$\epsilon$ = eddy diffusion coefficient
$\nu$ = kinematic viscosity
$\rho$ = density
$\tau_w$ = shear stress at wall

## LITERATURE CITED

1. Harriott, P., and R. M. Hamilton, *Chem. Eng. Sci.*, **20**, 1073 (1965).
2. Kishinevsky, M. K., T. B. Denisova, and V. A. Parmenov, *Inter. J. Heat Mass Transfer*, **9**, 1779 (1966).
3. Son J. S., and T. J. Hanratty, *AIChE J.*, **13**, 689 (1967).